10/8 Recitation

1. Let $V = \left\{ \begin{bmatrix} 2x - y + z \\ 3x + z \\ y - z \\ z \end{bmatrix} \,\middle|\, x, y, z \in \mathbb{R} \right\}$.

   (a) Show that V is a subspace of $\mathbb{R}^4$ by showing that it is the span of three vectors.

   (b) Interpret V as the column space of a matrix and use this matrix to determine $V^\perp$. Julia can be helpful here.

2. Let V be a subspace of $\mathbb{R}^m$. Show that $\dim V + \dim V^\perp = m$ by viewing V as the column space of a matrix and using facts you know about the four fundamental subspaces.

3. Let $W = \text{Span}\left\{ \begin{bmatrix} -1 \\ 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 2 \\ 1 \end{bmatrix} \right\}$ and let $\mathbf{b} = \begin{bmatrix} 0 \\ -1 \\ 4 \\ 2 \end{bmatrix}$

(a) Show that the three vectors in the description of W form a basis for W.

(b) Find the projection of $\mathbf{b}$ onto W and interpret your answer geometrically. Again, Julia can be very helpful here.

(c) What is the projection matrix P associated to this projection? Compute it using Julia and use it to verify (in Julia) your answer to part (b) is correct.

4. Continue working on the coding challenges in the **Julia8DotProducts** notebook.